Pix4D - Error Estimation

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1 3D Error estimation from tie points

$\mathbf{X} = (X, Y, Z)$	3D point in world coordinate system
$\mathbf{x}_i = (x_i, y_i)$	2D coordinate of the extracted or measured tie point in image i
$\mathbf{m}_i = (\tilde{x}_i, \tilde{y}_i)$	2D coordinate of the projected point in image i
$\mathbf{\Sigma}_i$	2D covariance of the tie point in image i
n	is the number of observations $\mathbf{x}_i$ of the point $\mathbf{X}$
m	is the number of variables to optimize, i.e. $m = 3$ for $\mathbf{X}$
$\mathbf{A}$	is a $(n \times m)$ matrix of partial derivatives with respect to the unknown parameters $\mathbf{X}$
$\mathbf{e}$	is the reprojection error vector of $\mathbf{X}$ in the images $\mathbf{x}_i$
S_0^2	variance component of the observations

A 3D point $\mathbf{X}$ is projected into the images by:

$$\mathbf{m}_i = \mathcal{P}_i(\mathbf{X}) \ , \tag{1}$$

where $\mathcal{P}_i$ contains the internal and external paramaters of the camera .
The covariance $\mathbf{\Sigma}_i$ of the tie points $\mathbf{x}_i$ are related to the extraction accuracy. They are displayed with yellow circles (the radius is up scaled for better visualization). The yellow cross shows the 2D location of the tie point $\mathbf{x}_i$, and the projected 3D point $\mathbf{m}_i$ in the image is shown in green.
The difference bettween the measured and projected 2D point constitutes the reprojection error. This difference is weighted by the 2D covariance $\mathbf{\Sigma}_i$ and is given for one image i by:

$$\mathbf{e}_i = (\mathbf{m}_i - \mathbf{x}_i)^T \mathbf{\Sigma}_i^{-1} (\mathbf{m}_i - \mathbf{x}_i) \ . \tag{2}$$

The complete estimation equation for a 3D point that is visible in n images is made of the sum of the individual components $\mathbf{e}$ in eqn.: 2:

$$\mathbf{e} = \sum_i^n (\mathbf{m}_i - \mathbf{x}_i)^T \mathbf{\Sigma}_i^{-1} (\mathbf{m}_i - \mathbf{x}_i) \ , \tag{3}$$

it is obtained by minimizing $\mathbf{e}$ as a function of the $3D$ point $\mathbf{X}$.

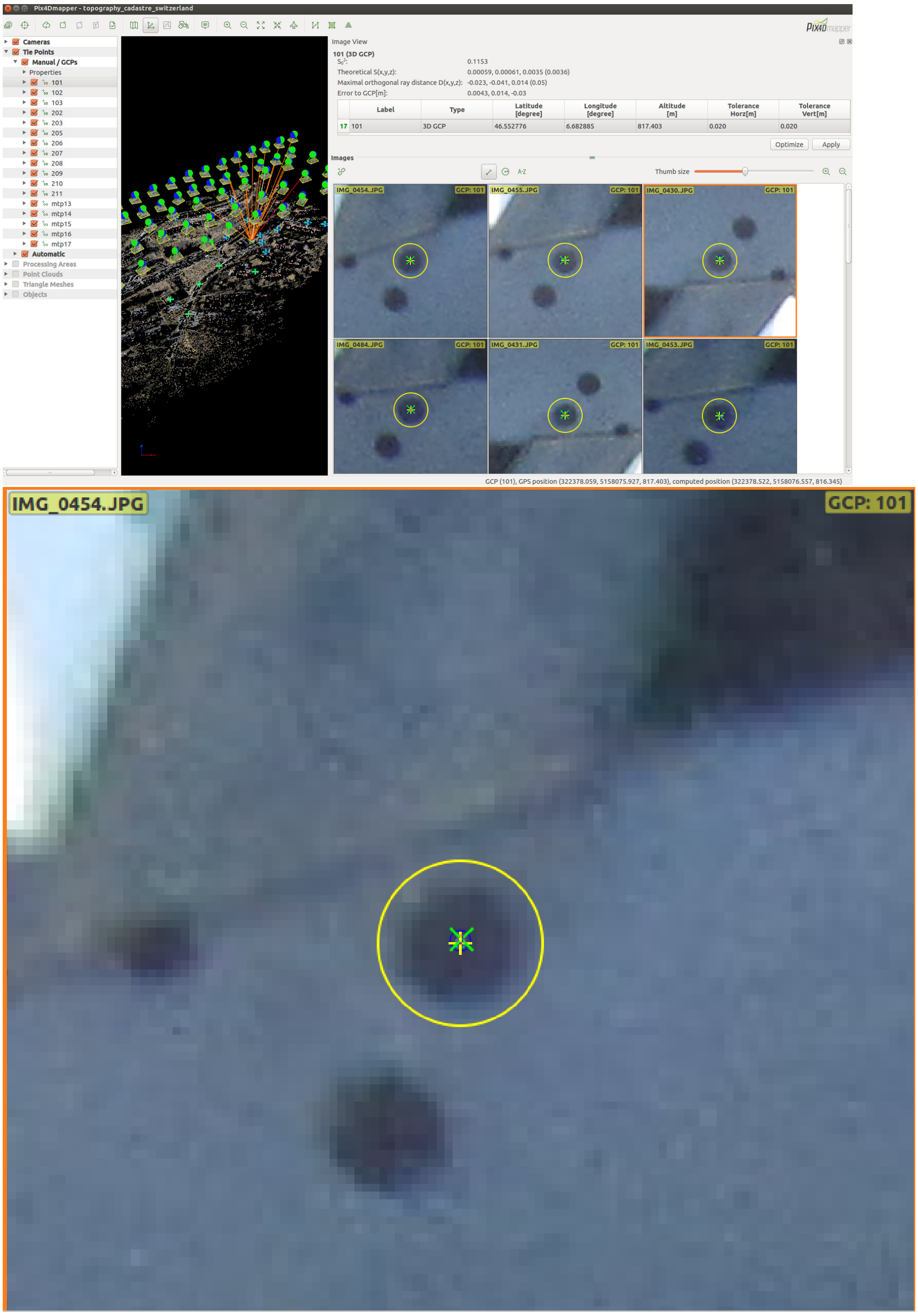

Figure 1: The figure shows an automatic extracted tie point which has been matched in three images (top) and the zoom to one tie point (bottom).

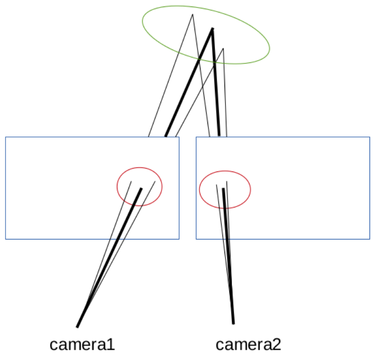

Figure 2: Sampled error estimation: Given the error ellipse of the individial keypoints (shown in red) the error ellipse (green) of the 3D point if computed by sampling keypoints with their error statistics and collecting the corresponding error statistics of the triangulated points.

1.1 Theoertical error estimation

The a posteriori variance component of all the observations for a given 3D point $\mathbf{X}$ is given by:

$$S_0^2 = \frac{\mathbf{e}^T \mathbf{e}}{n - m} \ , \tag{4}$$

where $n - m$ is the redundancy of the 3D point. Note that the minimal redundancy is required to be one, i.e. a 3D point $\mathbf{X}$ needs to be visible in at least two images ($n = 4, m = 3$).
The error of the estimated parameters $\mathbf{X}$ can be derivered from error propagation law are are given by:

$$\mathbf{S}_{theoretical}(\mathbf{X}) = S_0^2 \ tr((\mathbf{A}^T \mathbf{A})^{-1}) \ , \tag{5}$$

where $\mathbf{A}$ is the design matrix derivered from eq. 3.

1.2 Sampled error estimation

The error of the 3D point $\mathbf{X}$ is largly related to the accuracy $\mathbf{\Sigma}_i$ of the individial tie points $\mathbf{x}_i$. A smaller error ellipse $\mathbf{\Sigma}_i$ relates to smaller errors of $\mathbf{X}$. The sampled error of $\mathbf{X}$ is given by the error statistics of the minimum of eq. 2 by sampling the 2D tie point $\mathbf{x}_i$ according to its error ellipse $\mathbf{\Sigma}_i$. This error is computed by 100 samples and shown as $\mathbf{S}_{sampled}(\mathbf{X})$ in figure 2.